Length Effects due to Yarn Variability in Parallel-Lay Ropes

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#### ABSTRACT

Of all possible rope constructions, parallel-lay ropes offer the highest conversion efficiency from yarn properties to those of ropes; when made from high performance synthetic fibres they are being considered for many offshore and bridge structures.

There is very little interaction between the individual yarns; the rope can thus be seen as an aggregation of separate elements, and the strength and stiffness should be directly caculable from the yarn properties. However, the lack of interaction means that if a fibre breaks, its strength is lost over a considerable length of the rope, so fibre variability becomes much more important than in structured ropes.

This paper describes a rope analysis model for determining the strength and load-extension properties of parallel-lay ropes. The method proceeds from the properties of the individual yams to the properties of a short length of rope (the characteristic length), and then on to the properties of a long rope.

The effects of rope size, slack strain distribution and length are described in the paper, as well as considerations of the importance of termination reliability. It is shown that predictions can be made for long ropes, based on tests on short lengths of yarn.

## 1. INTRODUCTION

The move towards exploration for minerals and oil reserves in deeper water has meant that conventional offshore platforms founded directly on the sea-bed have had to be abandoned in favour of floating structures anchored over sub-sea facilities. A variety of mooring configurations are possible and most have similar requirements for the mooring ropes. They require high stiffness and low creep, to ensure accurate position-keeping; low weight for ease of installation and reduced top-side buoyancy requirement; and high durability<sup>1</sup>, <sup>2</sup>, <sup>3</sup>.

Parallel-lay ropes are ideal for these applications, particularly those made from low creep materials, such as polyesters or aramids. They are almost neutrally buoyant in water, which means that there is almost no change in the draught of the floating structure with depth. Installation schemes can also be devised that rely on floating into place moorings that have been assembled on or near the shore, complete with their anchoring systems.

In the parallel-lay rope the individual yarns are arranged parallel to the rope axis throughout the entire rope length. This parallel yarn path mobilises the full stiffness of the constituent fibres instead of introducing the additional extension of the helical or serpentine yarn paths found in typical twisted-lay (stranded) or braided rope constructions. In this parallel path construction there is a reduction in the ability to shed load from a broken fibre into its near neighbours, which in a structured rope is normally accomplished by friction between

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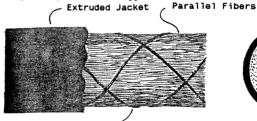
adjacent fibres.

It is to this effect that the present paper is addressed. In an ideal case, all fibres would have equal strength, identical load-extension characteristics, equal strain to break, and be of equal length and tension. However, fibres are inevitably variable, and rope construction techniques, however careful, will introduce some variation in the tension or slack in individual fibres. The variation in both fibre properties and fibre tension will mean that some fibres fail before others; load will then be shed to other fibres, which could then fail themselves.

A computational method is sought which will model the complete load deflection curve of the parallel-lay rope from first load to final rupture. The calculations are further complicated by the fact that the stress-strain curves of the fibres are non-linear, and also by the fact that the fibres cannot realistically be assumed to be completely independent over the (possibly very long) rope. Solutions to these problems will be addressed in detail in the paper.

### Parallel-lay rope construction

Parallel-lay ropes, such as Parafil manufactured by Linear Composites Ltd in the United Kingdom (Figure 1), contain parallel core yarns of fibres, typically, (but not necessarily) of either polyester or aramid fibres. These yarns are normally multi-filament yarns with little or no twist. The core of yarns is circular, and is contained within a sheath of thermoplastic material (typically a polyethylene). Other shapes, such as flat strip, are produced for applications such as soil reinforcement.





Fiber Binding Parallel Fibers
Figure 1. Parallel filament rope.
(Parafil - Linear Composites Ltd)

The rope is normally anchored by a spike and barrel termination, in which a central spike grips the fibres against a conical barrel (Figure 2). The taper angle of the barrel is small enough that the stresses generated do not cause transverse failure in the fibres, which are usually much stronger in the axial direction due to the highly oriented nature of the polymers. The spike and barrel arrangement ensures that every fibre is gripped equally, since each fibre is subjected to the same lateral pressure and can thus resist the load by friction equally well. This is in contrast to those systems which grip the fibres by external wedges, which tend to develop hoop compression in the outer layers of the rope, but do not grip the inner layers well.

There is inevitably some movement of the spike and yarns relative to the barrel on first loading, to allow the system to develop the necessary gripping forces, but observations on ropes subjected to a wide variety of tests show that the fibres all move uniformly<sup>5</sup>. Thus it will be possible, in the analysis which follows, to assume that the movement of the ends of the rope are transmitted directly to the yarns themselves, and that no relative movement between yarns occurs.

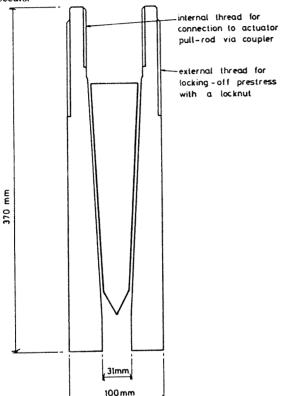


Figure 2. Typical anchorage for a 60 Tonne Parafil rope, (here modified for use as a prestressing tendon by the inclusion of both internal and external coupling threads). (From reference 5.)

A large number of tests have been carried out on Parafil ropes, and in virtually all of these tests, the rope has failed away from the termination<sup>5</sup>. The only cases where failure has occured at the termination in large ropes, have been where the spike has clearly not been placed centrally in the rope, so leading to an uneven sharing of the load across the rope. This would be avoided in ropes used for offshore applications, since some form of jig assembly of the termination would be required on the grounds of size alone.

In small ropes (with breaking loads less than about 10 Tonnes), failure at the termination is more common, since, perhaps paradoxically, it is more difficult to ensure even distribution of fibres around the spike<sup>6</sup>. When working out probabilities of failure, in later stages of this paper, it will be assumed that the termination can be made reliably in all practical cases. However, details will be given of how variation in terminal strength could be included in any analysis.

### Summary of model

The computer model adopted (Figure 3) represents a rope as a collection of N independent parallel elements; these may represent filaments, individual yarns, or groups of yarns. The

load-extension properties of each of these elements are chosen at random from a data base of typical yarn test results. Each of these elements is then assigned a randomly chosen slack value, to allow for variation in the make-up of the rope. The computer then sorts these elements on the basis of 'total strain to break' (i.e. element break strain from the database plus assumed slack strain) to find the order of element failure.

The model assumes that every yarn is subjected to the same strain and builds up a load extension curve for the whole rope. This is done by choosing increasing values of strain, and at each strain level, determining which elements are either slack, carrying load, or broken, and then calculating the total force carried in all yarns. The maximum load carried in the rope is noted for subsequent statistical analysis.

The process is repeated many times, each time randomly assigning load-elongation data from the data base and randomly choosing initial slack values for each element, to build up a distribution of failure loads for the rope, from which the probability of failure at any given load can be determined. The resulting mean value and distribution of failure loads represents the statistical properties of a short rope segment having no interaction between the parallel elements.

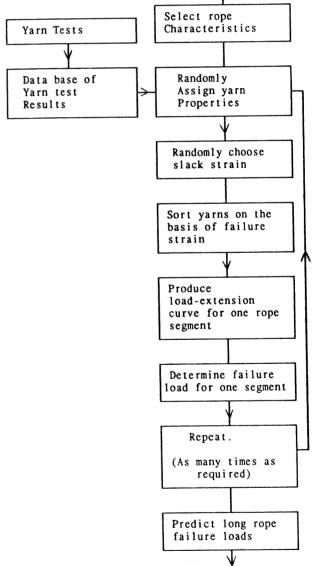


Figure 3. Outline of analysis model

In an actual parallel-lay rope, there is some interaction between the parallel yarn elements, such that a broken yarn has no effect beyond a distance which will be called the characteristic length. The failure probabilities found above will apply to each characteristic length, so that the total probability of failure over the whole length of the rope can be established on the assumption that the weakest characteristic length will fail first, and thus govern the strength of the whole rope. The various factors which govern this model will be discussed in more detail before presenting some results from the model.

## 2. DETAILED ASPECTS OF THE MODEL

# Choice of element - yarn or filament?

In the model as proposed, the term 'element' was used to describe the rope components. The rope is made from a number of multi-filament yarns, so the question arises as to whether the component element should be the yarns or the individual filaments. The strength of yarns is affected by the twist given to them prior to testing. As the twist angle increases, the strength of the yarn should drop as the square of the cosine of the twist angle, caused by the need to resolve forces into the axial direction of the yarn, but there is a slight initial increase in strength which is due to the transfer of load from broken filaments to neighbouring filaments through friction induced by the slight helix.

The post failure behaviour of yarms is also affected by twist. Tests on yarms with a high twist angle tend to show a rapid drop-off in load once the failure strain is reached; the helical winding ensures that load can build up again rapidly in a failed filament, so eventual failure occurs with all filaments breaking simultaneously. Similar tests on yarms with a low twist angle show a gradual drop in load, since failure in one yarm does not trigger failure in all the others. In many ways, low twist yarms represent on a small scale the problem that is being analysed here in ropes.

Parallel-lay ropes are often made from yarns with a small degree of twist; this is sufficient to cause some load transfer between filaments, but not high enough to cause a significant reduction in load capacity. This twist angle is effectively close to the border between the two post-failure yarn responses. When tested, most yarns will exhibit sudden failure, while some will exhibit more gradual failure.

This leaves open the question of whether yarms or filaments should be chosen as the basic element. In the present study, the yarm has been taken as the basic element, for a number of reasons.

1. There is at least some interaction between the filaments at the twist angles that are being considered. Thus, to work on the basis of filaments would require a two level model; the first stage to convert filament response to yarn response, and the second to relate yarn behaviour to rope behaviour. Rather than make assumptions about the way filaments are related to yarns, it is better to work with real yarn data in the first place.

 Yam data is more readily available than filament data, which requires the use of sensitive load measuring equipment and very careful handling.

3. The influence of post failure behaviour of the yams is not likely to cause a significant change in rope behaviour. Although there will be some difference between a yam carrying some load, and a yarn carrying none, the reduction in load from peak to zero still occurs over a short range of strains, even for zero twist yams. The effects of break strain variation and slack variation will be more significant.

 With a rope made from a large number of elements (yarns or filaments), the computer model will have to look at a representative sample, simply on the grounds of getting a solution in a reasonable time. This will introduce additional approximations into the analysis, but the degree of approximation will be lower, if the elements represent yarns, than if the elements represent filaments; the number of elements analysed will be a higher proportion of the total number of elements in the rope.

# Representation of yarn test data

The yam load-extension data, determined through testing, is represented in the computer model by expressing load for each yam as a fifth order polynomial of strain together with a break strain. Polyester and aramid yams cannot be modelled accurately by a linear function, and there are reductions in stiffness prior to failure in some yams of all materials. A polynomial of the fifth order has been found to be sufficiently complex to be able to pick up these responses. More complex polynomials could be used, but they suffer the problem of introducing spurious non-linearities into the response (which is why, for example, high order polynomials are not used for curve fitting for computer graphics programs, or as the basis for numerical integration algorithms).

The polynomial coefficients and the break strain can be obtained automatically by modifying standard data-logging programs for yarn test machines to incorporate a regression analysis for the unknown quantities. The break strain is defined as the strain at which the load drops significantly during a test. This could be dependent on the way the tests are carried out and the frequency with which the logging program samples the force, but in practice, a procedure by which the data logger notes a loss of 50% of the force between two successive strain readings has proved to work well.

#### Slack strain variation

The production procedures for parallel-lay ropes rely on bringing together a large number of yarns. These will be under some degree of tension, and this will, inevitably, vary between yarns. After manufacture, this tension will be relaxed, so that no yarns are in tension; clearly, those yarns that were under a lower tension during manufacture will now be slack.

The effect of yarn slack on rope properties is demonstrated later, but it is quite high, especially for high modulus fibres such as aramids. Obtaining slack data, however, is difficult; several methods are possible, but all suffer some problems.

 It should be possible to measure yarn tension during manufacture, but that is likely to be commercially sensitive information and unlikely to be released by the manufacturer.

2. A short length of a longer rope could be firmly clamped at each end and accurately cut through at right angles. The length of typical individual yarns could then be measured to determine the length (and hence slack) variation. This would have to be done accurately, however, since the variation in slack strain would need to be obtained with accuracy comparable to the variation of the break strain. Since Kevlar 49 yarns have a break strain of about 2%, this would imply that individual yarn length measurements would need to be accurate to about 0.1%.

 Individual yarns could be weighed and the length inferred from those measurements, but this presupposes no variation in the linear density of the yarns.

In the present model, the yarn slack strain will be represented by a mean value and a standard deviation, so that the effects of slack variation can be studied. In fact, the mean slack does not play any important role in determining the failure load in the rope, since it merely represents a strain offset on the rope force/rope strain curve. The variation in the slack strain, represented by the standard deviation of the slack, will be much more significant, as will be seen later. In practice, the value of the mean slack strain is usually taken as three times the standard deviation; this value being chosen from the properties of the normal distribution so that very few of the yams have a negative slack strain when randomly chosen values are selected. Tests on real ropes would allow more accurate values to be established.

### The Characteristic Length

The yarns are only held together at the ends, and if it is assumed that all yarns are straining uniformly along their length, there will be no relative movement between fibres during loading. However, when a yarn fails, the strain in that yarn (at least near the break) must reduce to zero, which means that there will be some relative movement between yarns. It is to be expected that there will be some resistance to this movement, either from friction effects, interlocking of fibres, or adhesion. At this stage, it is not necessary to discuss actual resistance mechanisms, but it is likely to be affected by such things as surface finish of the yarns, presence or absence of water, and lateral pressure.

Over a long length of rope, this resistance will accumulate to such an extent that the strain in the yarn will build up to the same level as that in the neighbouring yarns. It can be concluded that there will thus be a characteristic length (L\*); within that length, any yarn failure will cause a loss in strength over that length; outside that length, the yarn can be assumed to be fully effective.

The problem will arise of overlapping characteristic lengths. If it is assumed that yarn failures are randomly distributed over the whole length of the rope, then the lengths of weakened rope due to each yarn failure will overlap. Analysing such an arrangement would be very difficult. A detailed knowledge of the load transfer mechanism would be needed, taking into account the way load is carried, not just to the immediately adjacent yarns laterally (which then become the most likely ones to fail), but also to those more remote from the first failure.

A more tractable problem, and the one that will be tackled here, is to assume that the whole rope is divided up into a number of units, each of a length equal to the characteristic length. This is the so-called 'series-parallel model'? Within each unit, any broken yarn is assumed to have lost its strength over the whole of the unit, but to become fully effective again outside that unit.

# Determining the Characteristic Length

Although the concept of a characteristic length is valuable for visualising what is happening, ascribing values to L\* is not easy. Measuring the inter-yam forces directly is impossible, and finding a theoretical justification for those forces that would apply in all circumstances is also difficult. Nevertheless, the behaviour of the rope as a whole will be directly affected by the value of the characteristic length, as will be seen later, so a value for it will have to be obtained. There are a number of possible ways this could be done.

If a long rope ( $\triangle L^*$ ) is tested, it is to be expected that the rope will 'neck', with a concentration of yarn failures at one point due to the build-up in force in the remaining yarns at that point. In a short rope, ( $\angle L^*$ ), any yarn which fails will be ineffective over the whole length of the rope, so there will be no difference between the force in remaining yarns at other cross sections, and no tendency for failures to localise. Thus, by testing ropes of different lengths, and dissecting the broken

rope to determine the grouping of yarn failures, it should be possible to put upper and lower bounds on the characteristic length.

Tests carried out on 6 Tonne (13200 lb) Parafil ropes with Kevlar 49 core yarns and lengths of 2.9 metres and 10.9 metres showed these phenomena<sup>5</sup>. The sheaths of the longer ropes necked, with failures of the core yarns being concentrated at the neck into a length of about 250mm, while in the shorter lengths, no such necking was observed and yarn failures were evenly distributed over the length of the rope. This study was not carried out with a view to establishing L\*, but it does show bounds on its value in one situation.

Alternatively, if long ropes are tested to failure, and then taken apart, it is possible that some yarns will have failed in two places. If so, these must be at least half the characteristic length apart, since the force in the yarn must have built up to the breaking load away from the first yarn failure.

This last process can be tested directly, by measuring the force needed to extract a single yam from a short length of rope. If the rope is loosely fixed all along its length, and a single yam pulled out, the force necessary to extract the yam should vary with the length of the rope. By plotting the length of the rope against the extraction force, it is possible to extrapolate to find the length of rope needed to resist the breaking load of the yam.

Such tests have recently been undertaken on a pilot scale<sup>1</sup>, and the results are compatible with those noted earlier (Figure 4). On samples of the same 6 Tonne Parafil rope, it is predicted that a rope of length 3.1m would be needed to develop the full breaking load of the yarns, giving a characteristic length of 6.2m, which clearly lies between the 2.9m and 10.9m of the earlier study.

These values should not be taken as applicable to any rope other than the ones tested; as mentioned above, the results depend on many variables. Nevertheless, they confirm the general principles of the analysis that will be adopted.

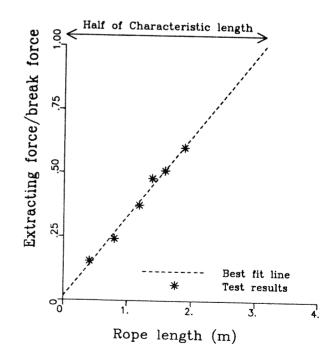


Figure 4. Force needed to pull a yarn from a parallel-lay rope, as a function of rope length (replotted from reference 8).

### Implementation of the model

The model is implemented in a computer program written in standard Fortran 77. The user enters data such as the number of yarns, slack variation, length of rope and characteristic length, which are normally input from the keyboard. The yarn test data is then read from a data file, and the program assignes to each yarn in the rope a load-extension curve from the data base. The program introduces further variability by assigning a slack to each yarn, chosen from a random distribution with the specified mean and standard deviation.

The rope strain at which each yarn will fail is then determined, and the yarns are sorted into break sequence using a Heap Sort algorithm. This step is not strictly necessary, but saves computer time since it avoids doing subsequent calculations on yarns that have already failed.

The model determines the force/strain response of the rope by taking successively increasing values of the strain, and calculating the force in each yarn from the strain and the polynomial representation of the load extension curve for the yarn. It is this part of the program which takes a significant time because of the need to evaluate a different polynomial for each yarn.

### 3. TYPICAL RESULTS

The rope force/rope strain results can be produced in tabular or graphical form. The results which follow are based on yarn data from tests carried out on a variety of Kevlar 49 yarns. The particular numbers given in the results should not therefore be regarded as applying to a specific rope, although the trends caused by varying slack, number of yarns and length are likely to be generally applicable.

Figure 5 shows a number of typical load/strain curves generated by the model for a rope with 100 yarns with a mean slack strain of 0.0045 and a standard deviation slack strain of 0.0015 (the average breaking strain of the yarns is 0.019, at a force of 215N (48.21bs)). The force scale is

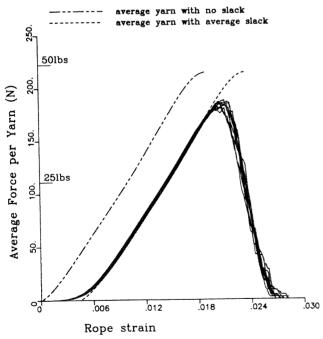


Figure 5. Predicted load extension curves for a parallel-lay rope (100 yarns, Kevlar 49, slack strain standard deviation 0.0015, mean 0.0045). Ten results superimposed.

expressed as the total force divided by the total number of yarns, so that easy comparisons can be made between ropes of different sizes. Two dashed curves are also shown, one showing the load/strain response of an average yarn with no slack, and one showing the response of an average yarn with average slack.

Ten different load/strain curves are shown on this plot, each corresponding to a different randomly chosen combination of yams and slacks from the same basic yam data. There is an initial reduction in rope stiffness as the slack in the different yams is taken up. After that, all the loading curves follow the behaviour of an average yarn with average slack, which is to be expected. After yams start to fail, the various curves diverge, as the randomness of the breaking strains starts to become apparent. This plot will be regarded as the datum; responses for other configurations should be compared with this plot.

## Effect of reducing slack

Figure 6 shows a similar plot for rope with the slack reduced by an order of magnitude, (to a standard deviation of 0.00015 and mean of 0.00045). There is a shift to the left, corresponding to the reduction in the mean slack strain, but the failure loads are higher, and the post-failure response steeper.

The effect of decreasing the slack is the same as decreasing the variability of the yarns; a lower variability will lead to an increase in strength, as predicted by standard bundle theory<sup>1</sup>. This is illustrated in Figure 7, which shows the total yarn strain variability for the basic yarn (with no slack), and the same yarns with slack strain standard deviations of 0.00015 and 0.0015; (the mean slack strain is three times the standard deviation).

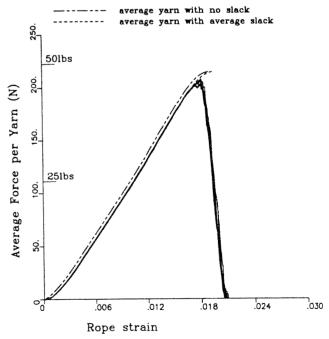


Figure 6. Predicted load extension curve for a parallel-lay rope (100 yarns, Kevlar 49, slack strain standard deviation 0.00015, mean 0.00045). Ten results superimposed.

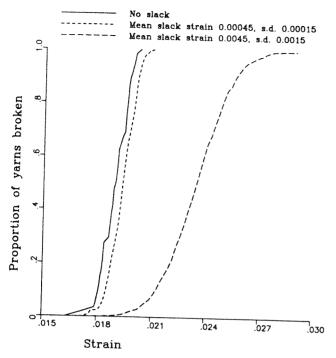


Figure 7. Cumulative distribution of yarn failures for different levels of slack (as generated randomly by the model).

# Effect of varying number of yarns

The effect of increasing the number of yams is shown in Figure 8, which shows the response of ropes with 1000 yams, and the same slack strain as the datum results in Figure 5. There is noticeably less variation between the different curves; indeed, although there are ten rope responses on the plot they are virtually indistinguishable.

There is a slight reduction in the average failure load, but a large reduction in the variation of the failure load, when compared with the smaller rope with the same slack variation. This will be significant when determining the effects of length.

# Effect of varying length

The model that has been developed can be regarded as applying to one characteristic length only. From the results of multiple rope analyses, the mean and standard deviation of the failure load for one characteristic length can be derived, and from these data the probability of failure (f(p)) at any load (p) can be deduced. This assumes that rope failures are normally distributed, an assumption that will need to be tested later, the same principles apply, however, no matter what distribution is used.

If the rope has a total length equivalent to a number of characteristic length segments,  $n.L^*$ , failure will be caused when the weakest characteristic length segment fails. From the statistics of the weakest links $^{1}$ , the total probability of failure of the rope at any given load will be  $(f(p))^n$ . The load corresponding to any desired probability can then be deduced to obtain predictions for the break load of a rope of any length.

The table below gives the results obtained from 20 analyses on the rope types illustrated earlier. In each case, the standard deviation of the slack strain is one third of the mean slack strain.

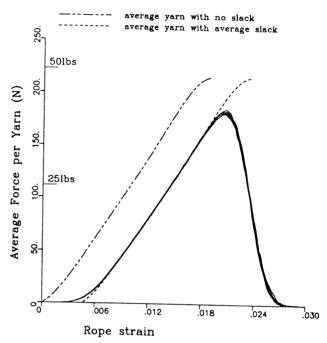


Figure 8. Predicted load extension curve for a parallel-lay rope (100 yarns, Kevlar 49, slack strain standard deviation 0.0015, mean 0.0045). Ten results superimposed.

Number of yarns	Mean slack strain	Failure load per yarn (N)	
		Mean	Stan'd Dev.
100	. 0045	184.118	2.584
100	. 00045	205.677	1.839
1000	. 0045	182.012	1.208

This data can be used to predict the effects of varying the length, as shown in Figure 9. Increasing the length of the rope reduces the strength, as would be expected, since it is more likely that a long rope will include a weak element. The larger rope (with 1000 yarns), is slightly weaker than the equivalent small rope (with 1000 yarns) for a single characteristic length. This larger rope loses strength less rapidly as the length increases, since it is less variable than the smaller rope.

There are implications here for the analysis of large ropes. If a rope has a very large number of yarns, it is likely that only a representative selection of yarns can be studied, due to limitations on computer capacity. The mean breaking load for a single element will not vary much if a small number of yarns (say 100) are studied. However, to determine the properties of a long rope, the variation in strength is also important; for that reason, as large a number of yarns as possible should be considered.

Any variation in anchorage efficiency, if less than 100%, could be included at this stage by introducing an additional probability of failure which was a function of the load level. If the probability of failure of one terminal, at a load (p), was g(p), then the total probability of failure of the rope with two terminals and a total length  $n.L^*$ , would be  $(f(p))^n.(g(p))^2$ . However, as stated in the introduction, the spike and barrel system used with Parafil can be designed to ensure that the termination can develop the full strength of the rope, and this refinement will not normally be needed.

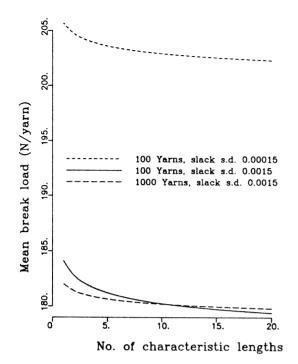


Figure 9. Effect of length variation on strength, for ropes analysed in Figures 5, 6 and 8.

## 4. CONCLUSIONS

A method has been presented for the analysis of parallel-lay ropes containing variable yarns. The implications of rope construction have been considered, as have the effects of inter-fibre friction. Multi-filament yarns are considered as the basic elements, rather than the individual filaments themselves.

If there is a large variation in yarn tensions, leading to a variation in yarn slack, then there is an initial reduction in rope stiffness as the yarn slack is taken up. Also, the resulting failure load of the rope is lower. A large variation in the load-extension characteristics of the yarns yields similar results. Increasing the number of elements (yarns) in the analysis decreases the variability. This has implications for the running time of the program.

The model calculates the performance of one characteristic length along the rope. Each particular parallel-lay rope design will have a characteristic length; failure of a yarn within a characteristic length will not cause a reduction in strength outside this length. The variability of strength for each characteristic length is then used as a basis of a calculation of strength of long ropes. The strength of the longer rope, containing many such characteristic lengths, will be less than that calculated for a single characteristic length.

### Acknowledgements

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