Shear analysis of concrete with brittle reinforcement

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SUMMARY
The design of steel-reinforced concrete relies on lower-bound plasticity theory, which allows an equilibrium-state to be postulated without considering compatibility. This is of particular benefit in shear design, due to the complexity of shear-transfer. However, lower-bound plasticity theory relies on stress-redistribution. If brittle reinforcement (such as FRP) is used in concrete, lower-bound plasticity theory cannot be applied. This paper examines compatibility, equilibrium and the material constitutive laws to establish the actual conditions within an FRP-reinforced beam subjected to shear. The implications of design based on lower-bound plasticity are highlighted, and a more fundamental approach to shear analysis is suggested.

LOWER-BOUND PLASTICITY THEORY
Our present understanding of shear in concrete is the result of a very large number of tests on steel-reinforced beams.[1,2] Shear theories evolved as their inadequacies became apparent, resulting in a variety of analysis techniques; for example, truss analogies, compression-field theory and the compressive force-path method.

Each shear theory assumes a different equilibrium-state within the beam; none is based on the actual stress distribution. Despite this, all the theories have been used safely to design steel-reinforced concrete. They rely on the *lower-bound* (or safe load) *theorem of plasticity*:

If any stress distribution throughout the structure can be found which is everywhere in equilibrium internally and balances certain external loads and at the same time does not violate the yield condition, those loads will be carried safely by the structure.[3]

The word “any” in this definition is most important, since it means that the designer does not need to know the actual stress distribution, which in many cases is anyway very difficult to determine.

Stress-redistribution
A structure does not know how it was designed, so the actual stress distribution at the working load may well not match the assumed equilibrium-state postulated without considering compatibility requirements. If the structure is ductile, *stress-redistribution* will occur when further increments of load are applied until the stress-distribution at failure matches that which was assumed in the design.

FRPRCS-5, Thomas Telford, London, 2001.

Concrete is a brittle material. It behaves in a quasi-ductile manner only under triaxial confinement.[4] In a traditional concrete beam, the arrangement of steel reinforcement confines the compression-zone concrete, and pseudo-plastic deformation can occur. However, compression-zone confinement is not guaranteed, particularly under shear loading, and with FRP reinforcement.

FRP reinforcement is certainly not ductile[5], although it may have a large strain capacity. Large-scale stress-redistribution cannot occur in an FRP-reinforced concrete beam, or in any structure with brittle reinforcement. Without stress-redistribution, lower-bound plasticity theory cannot be applied, and we cannot postulate an equilibrium-state without also satisfying compatibility requirements.

SHEAR IN A BEAM WITHOUT SHEAR REINFORCEMENT

Analysis of an FRP-reinforced concrete beam must be based on the actual stress-state, which satisfies compatibility and the material constitutive laws, as well as equilibrium.

Steel-reinforced concrete beams without shear reinforcement often fail in a brittle manner. Like FRP-reinforced concrete, lower-bound plasticity theory cannot be applied.[6] Numerous researchers have examined equilibrium and compatibility in beams without shear reinforcement, resulting in a reasonably detailed picture of the internal load-carrying mechanisms.[1] This is of great help when examining shear in FRP-reinforced beams.

This paper seeks to establish equilibrium and compatibility conditions only in general terms. More details can be found in the literature.[1,4,7] For brevity, beams with shear reinforcement are termed '*with stirrups*', while those without shear reinforcement are termed '*without stirrups*'.

Shear transfer mechanisms

Figure 1 is a schematic overview of a beam without stirrups, and the shear mechanisms acting in it. The details of these mechanisms will be discussed in subsequent sections. (To simplify discussion, only a 4-point, simply-supported beam is considered here).

For equilibrium in a shear-span, the moment must vary along the beam according to $V=\mathrm{d}M/\mathrm{d}x$. The moment can be represented by an internal force couple between the compression-zone concrete and flexural reinforcement actions. A change in moment, (thus shear transfer along the shear-span), is due to one of two mechanisms (*Figure 1*):

- variation of the magnitude of the internal actions;
- variation of the lever-arm between the actions.

Beam action

Beam action describes shear transfer by changes in the magnitude of the compression-zone concrete and flexural reinforcement forces, at constant lever-arm, requiring load-transfer between the two actions.[4]
In a cracked beam, load-transfer from the flexural reinforcement to the compression-zone occurs through the 'teeth' of concrete between cracks. Bending and failure of this concrete is studied by tooth models.[1] Bond between the concrete and reinforcement is also vital to beam action.

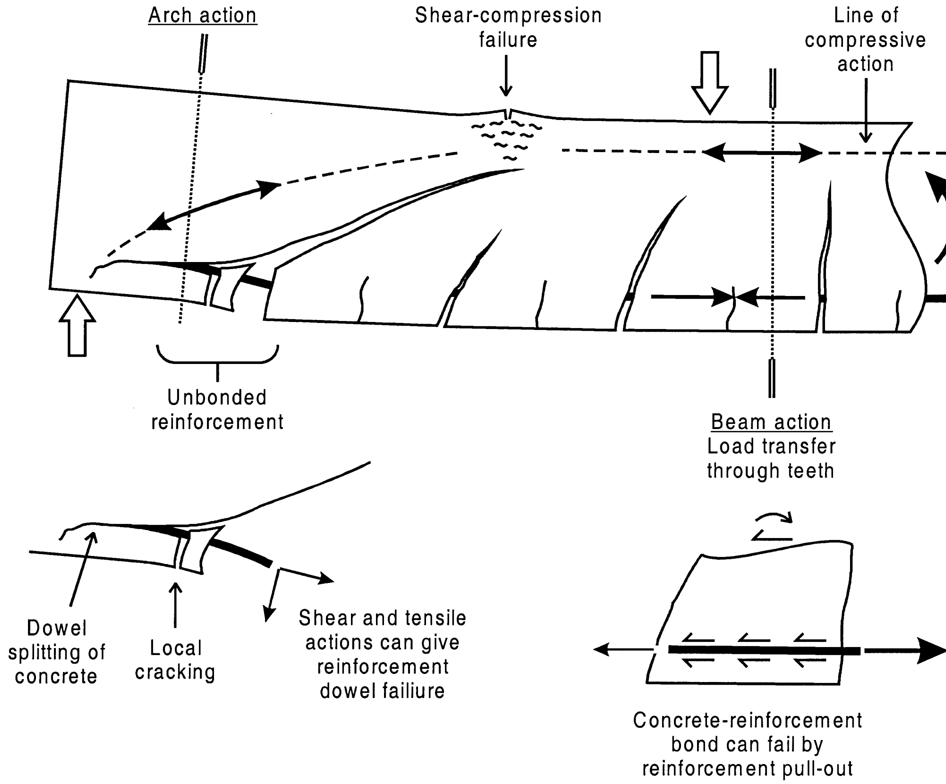

Figure 1 - Shear in a beam with no shear reinforcement.

Arch action
Arch action occurs in the uncracked concrete near the end of a beam, where load is carried from the compression-zone to the support by a compressive strut. The vertical component of this strut transfers shear to the support, while the constant horizontal component is reacted by the tensile flexural reinforcement. Both beam action and arch action can act in the same region.

Shear-compression theories study equilibrium and compatibility near the end of a beam, and across a single shear crack.[1] Recent shear-compression models (implemented by finite-element analysis) incorporate details of the reinforcement-concrete bond, tension-softening mechanisms across the crack, and detailed analysis of the compression-zone concrete.[8] Shear-compression analyses have also been applied to FRP-reinforced concrete.[9,10]

Shear failure

Figure 1 also shows the causes of shear failure:-

Crack propagation
The details of how arch and beam mechanisms act depend on inclined crack propagation through the beam.[4] Crack propagation is promoted by failure of other components of the beam. The cracks are a fundamental part of shear failure, but are rarely the sole cause.

Local cracking
Load transfer by concrete-reinforcement bond can cause localised cracking close to an existing crack.[11] This cracking increases the unbonded length of flexural reinforcement at the base of a crack, and consequently the reinforcement force drops. Crack propagation is required to bring the cracked section back into equilibrium.[7]

Dowel-splitting
A sudden increase in the unbonded length of reinforcement (and rapid crack propagation) also results from dowel-splitting.[7] This describes cracking of the concrete along the flexural reinforcement due to dowel action.[4]

Bond failure
Pull-out of reinforcement relative to the surrounding concrete can cause failure of bond at the interface (essential for beam action). This is most likely towards the centre of a beam.[4]

Reinforcement failure
Compatibility of the reinforcement across a crack is achieved by a combination of reinforcement stretching and slip of the reinforcement relative to the concrete.[7] With steel reinforcement, the slip is assumed to be negligible compared to plastic stretching. However, both stretching and slip are important for FRP reinforcement.

At the base of a shear crack, the reinforcement is subject to dowel (shearing) action. The low transverse strength of FRPs makes shear-tension failure likely before their pure tensile strength is achieved. This important failure mode is not observed in steel-reinforced beams.[12]

Shear-compression failure
Failure of the compression-zone can occur under the combination of shear and compressive axial actions. This "crushing" failure is due to the formation of tensile microcracks in the concrete. Microcrack formation is accompanied by volume dilation of the concrete, so that the strain-capacity of the compression-zone can be increased by local confinement (such as near a load application point, or by shear reinforcement).[4]

The triaxial stress-state in a compression-zone subjected to combined axial and shear loads remains difficult to model, especially above closely spaced cracks.

Diagonal-tension failure
If shear-compression failure is avoided, unstable crack propagation can split the compression-zone.

Predicting the shear failure load

Ideally, the shear-capacity of a beam could be predicted by detailed examination of the shear transfer mechanisms, crack propagation and component failures. However, further research is required before this is possible. The compression-zone concrete, and dowel splitting along the reinforcement, are particularly difficult to model.[7]

In design codes,[13,14,15] the shear-capacity of a beam without stirrups is found empirically, based on the load at which the first shear crack forms. For beams with long shear-spans, shear cracks form due to a sudden increase in the unbonded length of flexural reinforcement. Rapid crack propagation follows, and the first cracking load is equal to the shear-capacity. For beams with short shear-spans, crack propagation may be more gradual, and the ultimate shear load can be considerably higher than that predicted by the codes.[4]

SHEAR IN A BEAM WITH SHEAR REINFORCEMENT

Shear reinforcement is used to make a beam fail in flexure rather than shear. As in beams without stirrups, equilibrium and compatibility must be satisfied by examination of arch and beam actions, crack propagation, and component failure. However, the details of shear transfer are different, due to the tensile actions carried across cracks by the shear reinforcement:

- Shear reinforcement confines the compression-zone concrete, and thus increases its shear-capacity.
- Shear reinforcement encloses the flexural reinforcement and can prevent dowel-splitting of the concrete. However, dowel-failure of FRP reinforcement is promoted.
- For a given applied load, equilibrium of a cracked section with stirrups requires a shorter crack length, but larger crack opening width, than one without stirrups. The shape of the crack will also be different.
- Concrete softening mechanisms are less effective across a wider crack: if the surfaces of a crack are completely separate, aggregate interlock cannot occur.[4]

Shear transfer in beams with stirrups has not been examined in as much detail as that in beams without stirrups. In steel-reinforced concrete, researchers have been able to take advantage of stress-redistribution (afforded by the yielding stirrups), and apply lower-bound plasticity theory.

Superposition of the "concrete" and "stirrup contributions"

An underlying assumption of many shear analyses is that the "concrete contribution" (V_c) and "stirrup contribution" (V_s) can be superposed to give the net shear-capacity of a beam:

$$V = V_c + V_s$$

The "concrete contribution" is the shear-capacity of an equivalent beam without stirrups. However, in a beam *with* stirrups, the shear carried by the concrete will be quite different to that carried by a beam *without* stirrups. The stirrups restrain crack growth, the strain-capacity of the compression-zone is increased by confinement, and although dowel-splitting is prevented, dowel failure of FRP reinforcement is promoted (potentially at a lower load).

The "stirrup contribution" is the capacity of the stirrups acting on their own, usually calculated by assuming that all the shear reinforcement yields. (Not all the stirrups are likely to yield, especially close to the crack tip).

The "concrete contribution" and "stirrup contribution" systems are considered separately. Compatibility of the shear reinforcement with the rest of the beam is not examined, so the superposition relies on stress-redistribution.

Compressive-force-path method

The compressive-force-path method[4] is based on a more realistic assessment of the capacity of a beam without stirrups than currently used in the codes (but remains empirical). Shear reinforcement is placed to prevent propagation of the critical shear crack, and is assumed to yield. The net shear-capacity found by superposition, thus relying on stress-redistribution.

Compression-field theory

Compression-field theory is based on the biaxial response of square elements of steel-reinforced concrete. The original constitutive relationships were analytical, but these were replaced by more realistic empirical equations.[16] A small number of tests have been carried out to establish equivalent constitutive equations for FRP-reinforced elements.[17,18] A different constitutive relationship is likely to be needed for each type of FRP, due to the considerable variation in reinforcement properties.

If the element is considered in isolation, the use of empirical constitutive relationships avoids assumptions about the internal equilibrium-state. However, if the element is part of a beam, simplifications must be made that rely on stress-redistribution. For example, a uniform shear stress is assumed through the depth of the beam.[2] Furthermore, the shear-capacity is calculated on a critical vertical section. In the shear-span of a point-loaded beam (where the shear force is constant), this implies that all sections in the shear-span are critical. In reality, failure is due to a single critical crack.

Truss analogies

The truss analogies are most commonly used in design. The assumed internal equilibrium-state comprises tensile shear reinforcement and inclined compressive struts of concrete.

The original, *Mörsch truss analogy*[1] uses a 45° strut angle, and predicts failure when the shear reinforcement yields. The codes combine this with the "concrete contribution" to give the total shear-capacity.[13,14,15]

The *modified truss analogy*[15,19] establishes an optimal lower-bound for the shear-capacity by varying the compressive strut angle to give reinforcement yield and web concrete failure simultaneously. Plasticity theory is used explicitly.

Both truss analogies rely on stress-redistribution from the postulated truss mechanism to the actual equilibrium-state. The truss mechanism is not observed experimentally: the assumed compressive struts would have to cross curved cracks in the shear-span, even though the crack surfaces are completely separate.[4] Furthermore, the truss analogies are sectional design methods (as for compression-field theory), and shear is not a sectional failure.

CURRENT PROPOSALS FOR SHEAR DESIGN WITH FRP REINFORCEMENT

The danger of using lower-bound plasticity theory for shear design with brittle reinforcement has been noted before.[5,20] Despite this, the proposed shear design clauses[21,22,23] for FRP-reinforced concrete reflect their steel-reinforced origins, and are based on truss analogies.

The modern understanding of equilibrium and compatibility conditions in a beam without stirrups allows specific concerns to be identified with the code proposals.

The "concrete contribution"

FRP code proposals for the "concrete contribution"

The current proposals for FRP-reinforced concrete shear design take the "concrete contribution" for steel-reinforcement and modify it by the ratio of the stiffness of steel to FRP.[24] The stiffness of the reinforcement certainly affects the shear-capacity of the beam, but is only one of the parameters that changes when steel reinforcement is replaced by FRP. It has not been established that it is the most important parameter.

The load at first shear crack formation

The "concrete contribution" in design codes is the load at which the first shear crack forms in a beam without stirrups. As discussed above, the empirical code expressions are derived from tests on beams with long shear-spans, in which a sudden increase in the unbonded length of reinforcement leads to rapid shear crack propagation. This increase in unbonded reinforcement length may be due to dowel-splitting of the concrete, or local cracking due to load-transfer across the concrete-reinforcement interface (*Figure 1*).

Anisotropy gives FRPs a low transverse stiffness, so that the dowel-splitting load will be different to that with steel reinforcement. Similarly, concrete-steel bond is very different to concrete-FRP bond. Furthermore, bond characteristics can vary greatly between different types of FRP reinforcement. The stiffness of the reinforcement is thus not sufficient on its own to describe the load at which the first shear crack forms.

The actual "concrete contribution"

In beams with shear reinforcement, the actual "concrete contribution" is the shear carried by all the beam components *except* the shear reinforcement. As already stated, the "concrete contribution" in a beam with stirrups is different to the shear-capacity of a beam without stirrups.

The "concrete contribution" can only be found by considering compatibility and equilibrium conditions at failure. A simple comparison of FRP-reinforced and steel-reinforced sections can be made by examining a single shear-crack (*Figure 2*).

An FRP-reinforced section will probably be less stiff than a steel-reinforced section. FRP reinforcement can be less stiff than steel, and there is typically more slip between FRP reinforcement and concrete. Furthermore, if the design takes advantage of the high ultimate strength of FRP reinforcement, a smaller flexural reinforcement ratio may be used. The reduced section stiffness with FRP reinforcement leads to wider crack opening.

In steel-reinforced concrete, there is evidence that aggregate interlock is negligible.[4] In FRP-reinforced beams, the wider cracks mean that aggregate interlock is very unlikely to act.[5,20]

Thus, the only "concrete contribution" shear-transfer mechanism is in the compression-zone. Wider crack openings require increased compression-zone deformation. It has already been seen that the compression-zone is difficult to analyse (since triaxial confinement is important), but there is no reason to expect that the deformation-capacity of the compression-zone in an FRP-reinforced beam is greater than that in a steel-reinforced beam.

Finally, reinforcement dowel-failure is important in FRP-reinforced beams. The "concrete contribution" in an FRP-reinforced beam (where dowel failure occurs) could be significantly lower than that in a steel-reinforced beam (where dowel failure does not occur).

The "stirrup contribution"

Shear reinforcement must be effective at small crack openings, to restrain crack propagation. However, it must not fail at a large crack opening. With steel reinforcement, lower-bound plasticity theory allows us to assume that all the reinforcement yields along a crack, and both criteria can be satisfied.

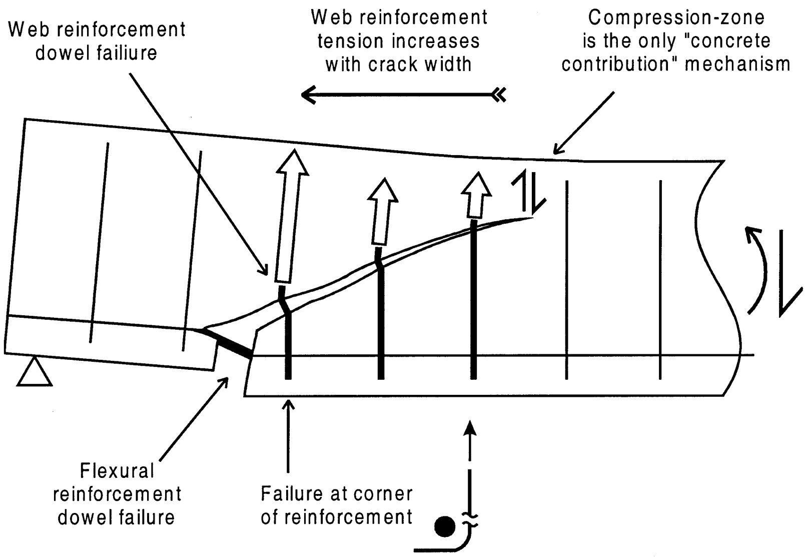

Figure 2 - Shear in a beam with shear reinforcement.

The code proposals for FRP reinforcement specify a limiting stirrup strain.[24] The proposals are based on truss analogies that assume a constant reinforcement strain along a crack (they are sectional design methods). However, it has been observed experimentally that the strain in FRP shear reinforcement varies along a crack.[25] This is to be expected, since the crack width varies along its length (as shown in *Figure 2*).

The shear reinforcement capacity (the "stirrup contribution") is the shear carried across a crack by the stirrups just before the first stirrup fails. Failure of FRP shear reinforcement can be due to either (a) dowel action rupture, or (b) a stress concentration at a corner in the reinforcement. In either case, the stirrup closest to the base of the crack is likely to fail first (*Figure 2*). The tension in the remaining stirrups just before failure depends on the crack geometry, reinforcement stiffness, bond characteristics, and localised concrete failure.[7] There is no reason to suppose that a uniform limiting strain can be applied to find the net shear carried across a crack.

CONCLUSIONS

Designers must recognise when they rely on lower-bound plasticity theory

Lower-bound plasticity theory is invoked whenever an assumption is made about equilibrium conditions in a beam. In beams with brittle reinforcement, large-scale stress redistribution (required, for example, by the truss analogies) is not possible. Small-scale stress redistribution is probably possible, and this is necessary to allow for uncertainties in the material constitutive laws.

Practising engineers are used to relying on lower-bound plasticity theory for steel-reinforced concrete design, even if they do not realise it. They are unlikely to appreciate the implications of using brittle reinforcement. Designers must be aware that the "safety-net" of lower-bound plasticity theory does not exist when using FRP reinforcement.

A realistic approach to shear design with brittle reinforcement
The current proposals for shear design with FRP reinforcement have been adopted in the absence of a more rational analysis. However, they rely on stress-redistribution, which cannot occur in an FRP-reinforced beam.

A realistic model for shear in FRP-reinforced concrete must be based on a fundamental examination of equilibrium, compatibility and the material constitutive laws in a beam. The modern understanding of shear in steel-reinforced concrete beams without stirrups is based on a very similar approach, and the techniques developed for those beams can be extended to analyse beams with FRP reinforcement.

The framework of a realistic model is developed elsewhere.[7,26] Further research is required to complete the model, particularly concerning the compression-zone concrete and dowel-splitting failure. This research is fundamental to the shear design of concrete with brittle reinforcement, but would also be of benefit for steel-reinforced concrete.

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